Lecture 4 – Exercises Solutions

Exercise 1: Convective heat transfer by forced convection [L4, slides 5-6, 11-12]

As air flow is elevated (4.6 m/s), forced convection along the wall is expected. Thus, a correlation Eqn. (4-13) for a horizontal plate should be used. To determine the flow regime, $\boldsymbol{Re}$ needs to be determined (characteristic dimension L_c=2 m, width of the pavement) for properties estimated at $T = \frac{T_s+T_f}{2}$ => 52.5°C.

Per Eqn. (4-11b): $Re = \frac{\rho \cdot U \cdot L_c}{\mu} = \frac{1.084 \cdot 4.6 \cdot 2}{19.75 \times 10^{-6}} = \mathbf{5.05 \times 10^5} > 5 \cdot 10^5$ – flow is turbulent

Heat transfer coefficient for turbulent forced convection over a horizontal plate:

$$\boldsymbol{Pr} = \frac{v}{\alpha} = \frac{\mu}{\rho \cdot \alpha} = \frac{19.75 \times 10^{-6}}{1.084 \cdot 25.89 \times 10^{-6}} = 0.704$$

$$\boldsymbol{Nu} = 0.037 \cdot Re_L^{0.8} \cdot Pr^{\frac{1}{3}} = 0.037 * (5.05 \times 10^5)^{0.8} \cdot 0.704^{\frac{1}{3}} = \mathbf{1202}$$

$$\boldsymbol{h_{conv}} = \frac{Nu \cdot k}{L_c} = \frac{1202 \cdot 0.0283}{2} = \mathbf{17}\ \frac{W}{m^2 \cdot K}$$

Finally, the rate of heat transfer from the hot pavement surface per Eqn. (4-1) is:

$$\dot{\boldsymbol{q}}_{\boldsymbol{conv}} = h_{conv} \cdot (T_s - T_f) = 17 \cdot (70 - 35) = \mathbf{595}\ \frac{W}{m^2}$$

The rate of heat removal when airflow is turbulent is substantial, comparable with the incident solar flux.

Exercise 2: Combined modes of heat transfer (convection and radiation) [L4, slides 9-12, 14-15]

I. Analysis of the indoor side:

(a) Convective heat transfer coefficient:

- As airflow is negligible, free convection is expected along the wall; thus, a correlation Eqn. (4-12) for a vertical plate should be used.
- To determine the flow regime, $\boldsymbol{Ra_H}$ needs to be determined per Eqn. (4-10) (characteristic dimension is the height of the room H=2.5 m).
- Note that temperature difference ΔT used in Gr number is $(T_f - T_s)$ as surface temperature $T_s = 273.15 + t_{si}$ is cooler than the air temperature $T_f = 273.15 + t_i$.
- Air properties should be determined at average temperature $T = \frac{T_s+T_f}{2} = 294.15\ K$ => 21°C.
- As the kinematic viscosity of air is not given, it needs to be determined first per Eqn. 4.5:

$$v = \frac{\boldsymbol{\mu}}{\boldsymbol{\rho}} = \frac{8.26 \times 10^{-6}}{1.2} = 15.22 \times 10^{-6} \frac{m^2}{s}$$

$$Ra_H = Gr \cdot Pr = \frac{g \cdot \beta \cdot (T_f - T_s) \cdot H^3}{v^2} \cdot \frac{v}{\alpha} = \frac{g \cdot \frac{1}{(T_s+T_f)/2} \cdot (T_f - T_s) \cdot H^3}{v^2} \cdot \frac{v}{\alpha} = \frac{9.81 \cdot \frac{1}{294.15} \cdot (2) \cdot 2.5^3}{(15.22 \times 10^{-6})^2} \cdot \frac{15.22 \times 10^{-6}}{21.49 \times 10^{-6}} =>$$

$\boldsymbol{Ra_H = 3.18 \times 10^9} > 10^9$ – flow is turbulent

Dimensionless heat transfer coefficient for turbulent free convection over a vertical plate:

$$\boldsymbol{Nu} = 0.1 \cdot Ra_H^{\frac{1}{3}} = 0.1 * 3.18 \times 10^9 = \boldsymbol{147}$$

$$\boldsymbol{h_{conv,i}} = \frac{Nu \cdot k}{L_c} = \frac{Nu \cdot k}{H} = \frac{147 \cdot 0.02595}{2.5} = \boldsymbol{1.52}\ \frac{W}{m^2 \cdot K}$$

$$\boldsymbol{R_{c,i}} = \frac{1}{h_{conv,i}} = \frac{1}{1.52} = \boldsymbol{0.655} \frac{m^2 \cdot K}{W}$$

(b) Radiative heat transfer coefficient:

Per Eqn. (3-19) from Lect. 3: $h_{rad} = \varepsilon \cdot \sigma \cdot (T_{si}^2 + T_i^2) \cdot (T_{si} + T_i)$

$$\boldsymbol{h_{rad,i}} = 0.9 \cdot 5.67 \cdot 10^{-8} \cdot (293.15^2 + 295.15^2) \cdot (293.15 + 295.15) = \boldsymbol{5.19} \frac{W}{m^2 \cdot K}$$

$$\boldsymbol{R_{r,i}} = \frac{1}{h_{rad,i}} = \frac{1}{5.19} = \boldsymbol{0.193}\ \frac{m^2 \cdot K}{W}$$

II. Analysis of the outdoor side:

(a) Convective heat transfer coefficient:

- As airflow is substantial (U=1.2 m/s), forced convection along the wall is expected. Thus, a correlation Eqn. (4-13) for a horizontal plate should be used (the wind is tangential; thus, the wall can be considered horizontal).
- To determine the flow regime, $\boldsymbol{Re_L}$ needs to be determined (characteristic dimension L_c=6 m) for properties estimated at $T = \frac{T_s+T_f}{2} => -2.5$°C.

Per Eqn. (4-11b): $Re = \frac{\rho \cdot U \cdot L_c}{\mu} = \frac{1.305 \cdot 1.2 \cdot 5}{17.09 \times 10^{-6}} = \boldsymbol{4.58 \times 10^5} < 5 \cdot 10^5$ – flow is laminar

Dimensionless heat transfer coefficient for laminar forced convection over a horizontal plate:

$$\boldsymbol{Pr} = \frac{v}{\alpha} = \frac{\mu}{\rho \cdot \alpha} = \frac{17.09 \times 10^{-6}}{1.305 \cdot 18.42 \times 10^{-6}} = 0.71$$

$$\boldsymbol{Nu} = 0.664 \cdot Re_L^{\frac{1}{2}} \cdot Pr^{\frac{1}{3}} = 0.664 * (4.58 \times 10^5)^{\frac{1}{2}} \cdot 0.71^{\frac{1}{3}} = \boldsymbol{400.9}$$

$$\boldsymbol{h_{conv,e}} = \frac{Nu \cdot k}{L_c} = \frac{400.9 \cdot 0.02417}{5} = \boldsymbol{1.94}\ \frac{W}{m^2 \cdot K}$$

$$\boldsymbol{R_{c,e}} = \frac{1}{\boldsymbol{h_{conv,e}}} = \frac{1}{1.94} = \boldsymbol{0.516} \frac{m^2 \cdot K}{W}$$

(b) Radiative heat transfer coefficient:

Per Eqn. (3-19) from Lect. 3: $h_{rad} = \varepsilon \cdot \sigma \cdot (T_{se}^2 + T_{sky}^2) \cdot (T_{se} + T_{sky})$

$$\boldsymbol{h_{rad,e}} = 0.9 \cdot 5.67 \cdot 10^{-8} \cdot (273.15^2 + 262.15^2) \cdot (273.15 + 262.15) = \boldsymbol{3.91} \frac{W}{m^2 \cdot K}$$

$$\boldsymbol{R_{r,e}} = \frac{1}{h_{rad,e}} = \frac{1}{3.91} = \mathbf{0.255}\ \frac{m^2 \cdot K}{W}$$

III. Comparison:

- Interestingly, dimensionless heat transfer coefficient $\boldsymbol{Nu}$ for forced laminar convection over an outdoor surface is almost 2.7 times greater than for turbulent natural convection at the indoor surface; however, dimensional heat transfer coefficients $\boldsymbol{h_{conv}}$ are relatively comparable.
- Radiative heat transfer coefficient $\boldsymbol{h_{rad}}$ from the interior wall is **1.3 times** greater than from the exterior wall. The higher the temperature of the bodies exchanging heat (given the same emissivity), the higher the heat transfer coefficient.
- Comparison of combined thermal resistances:

$$\boldsymbol{R_{se}} = \frac{1}{\frac{1}{R_{r,e}}+\frac{1}{R_{c,e}}} = \frac{1}{\frac{1}{0.255}+\frac{1}{0.516}} = \mathbf{0.17}\ \frac{W}{m^2 \cdot K} \qquad \boldsymbol{R_{si}} = \frac{1}{\frac{1}{R_{r,i}}+\frac{1}{R_{c,i}}} = \frac{1}{\frac{1}{0.193}+\frac{1}{0.655}} = \mathbf{0.15}\ \frac{W}{m^2 \cdot K}$$

Surface thermal resistances $\boldsymbol{R_{se}}$ and $\boldsymbol{R_{si}}$ are comparable with the conductive thermal resistance of $\boldsymbol{R_{k,tot}} = \mathbf{0.166}\ \frac{W}{m^2 \cdot K}$, slightly lower for the interior surface.

Exercise 3: Evaporation [L4, slides 25-26]

1. Name the main drivers of evaporation in the summertime. Would the evaporation be strong in Chicago? Comment on your results considering the Bowen ratio.

The three main drivers of evaporation are:

1) *Supply of thermal energy from solar radiation:* Because of its latitude, Chicago receives enough solar radiation in the summertime.
2) *Presence of wind, a mechanism for transferring water vapor away from the surface:* Given its location next to the lake, Chicago has a regular breeze from the lake. The downtown area of Chicago forms a wind barrier that prevents wind from entering the city. However, the rest of the city is entirely flat, and the lake breeze can penetrate deep into the city.
3) *A supply of water for evaporation:* large parks and the lake are apparent water sources in the city.

As the three main drivers of evaporation are present in Chicago, we can expect that evaporation is substantial in Chicago. This notion is supported by the value of the Bowen ratio of 1.08, as the value close to 1 indicates that *latent* and *sensible heat* play an equal role in the city, and surfaces are more or less in a thermal balance with their atmosphere. In contrast, the Bowen ratio for other cities is 1.8 for Tokyo, 2.55 for Basel, and 5.42 for Vancouver (see slide 25 and Oke, p. 160).

2. Calculate the *potential evaporation rate* and the *latent heat* using the Penman method, considering that the ground heat flux Q_G is negligible.

The Penman method is expressed as the sum of two components, per Eqn. (4-27). Thus, each of them needs to be found first considering the following m and γ parameters:

$$\boldsymbol{m} = 4098 \cdot \frac{0.6108 \cdot \exp\left(\frac{17.27 \cdot t_r}{t_r+237.3}\right)}{(t_r+237.3)^2} = 4098 \cdot \frac{0.6108 \cdot \exp\left(\frac{17.27 \cdot 10.2}{10.2+237.3}\right)}{(10.2+237.3)^2} = 0.0833\,\frac{kPa}{K}$$

$$\boldsymbol{\gamma} = \frac{c_p \cdot p_a}{0.622 \cdot L_v} = \frac{1005 \cdot 1.01 \times 10^5}{0.622 \cdot 2477 \times 10^3} = 67.2\ Pa/K = 0.0672\ \frac{kPa}{K}$$

(a) The radiation term (Eqn. 4-27):

$$\boldsymbol{E_S} = \frac{m}{m+\gamma}\left(\frac{Q^* - Q_G}{L_v}\right) = \frac{0.0833}{0.0833 + 0.0672} \times \frac{545 - 0}{2477 \times 10^3} = 12.18 \times 10^{-5}\ kg/(m^2 \cdot s)$$

(b) For the aerodynamic term, the *actual* and *saturation water vapor pressure* must be computed first to calculate the *actual* and *saturated specific humidity* using Eqn. (2-6a), (2-10a) from Lect. 2.

Eqn. (2-6a) → $p_{v,sat} = 611 \cdot e^{\frac{(17.08 * t)}{(234.18+t)}} = 611 \cdot e^{\frac{(17.08*10.2)}{(234.18+10.2)}} = 1246.4\ Pa$

Eqn. (2-10a) → $p_v = \varphi \cdot p_{v,sat} = 0.71 \cdot 1246.4\ = 884.9\ Pa$

Specific humidity at saturation conditions is determined using Eqn. (2-8a):

$q_s = 0.622 \frac{p_{v,sat}}{p - 0.378 \cdot p_{v,sat}} = 0.622 \frac{1246.4}{1.01 \cdot 10^5 - 0.378 \cdot 1246.4} = 0.00771 \frac{kg}{kg} = 7.71\ g/kg$

Actual specific humidity using Eqn. (2-8a):

$q = 0.622 \frac{p_v}{p_a - 0.378 \cdot p_v} = 0.622 \frac{884.9}{1.01 \cdot 10^5 - 0.378 \cdot 884.9} = 0.00547 \frac{kg}{kg} = 5.47\ g/kg$

The drying power of air (Eqn. 4-20b):

$E_a = \rho \cdot C_W \cdot U \cdot (q_s - q) = 2.45 \cdot 1.32 \times 10^{-3} \cdot 4.5 \cdot (7.71 - 5.47) \times 10^{-3} =$
$0.0326\ \times 10^{-3}\ kg/m^2 \cdot s$

Finally, the *aerodynamic* potential evaporation rate per Eqn. (4-27) can be calculated as:

$\boldsymbol{E_T} = \frac{\gamma}{m+\gamma} E_a = \frac{0.0672}{0.0833+0.0672} \cdot 0.0326\ \times 10^{-3} = 0.0146\ \times 10^{-3}\ kg/m^2 \cdot s = 1.46$
$\times 10^{-5}\ kg/(m^2 \cdot s)$

The *aerodynamic evaporation term* $\boldsymbol{E_T}$ is smaller than *the radiation evaporation term* $\boldsymbol{E_S}$.

(a) The **potential evaporation rate** calculated per Penman equation is:

$\boldsymbol{E_{pot}} = E_S + E_T = 13.64 \times 10^{-5}\ kg/m^2 \cdot s$

3. **Determine the *latent heat flux* $\boldsymbol{Q_E}$ and *sensible heat flux* $\boldsymbol{Q_H}$ using the surface energy balance method.**

Using Eqn. (4-25) → $\boldsymbol{Q_E} = \frac{Q^* - Q_G}{1+B} = \frac{545-0}{1+1.08} = 262\ W/m^2$

Using Eqn. (4-26) → $\boldsymbol{Q_H} = B \cdot \frac{Q^* - Q_G}{1+B} = 1.08 \cdot \frac{545-0}{1+1.08} = 283\ W/m^2$

4. **Compare the actual evaporation rate determined by knowing $\boldsymbol{Q_E}$ in (3) with the potential evaporation rate determined in (2).**

Using Eqn. (1-6) → $Q_E = L_v \cdot E$ → $\boldsymbol{E} = \frac{Q_E}{L_v} = \frac{262}{2477 \cdot 10^3} = 0.106\ \times 10^{-3}\ kg/m^2 \cdot s$

The maximum possible potential evaporation rate ($13.64 \times 10^{-5}\ kg/m^2 \cdot s$) is greater than the actual rate ($10.6\ \times 10^{-5}\ kg/m^2 \cdot s$).